METHOD OF CALCULATING FLOW DISTRIBUTION ALONG CONTACT, FILTER, AND SIMILAR APPARATUS OF CYLINDRICAL SHAPE

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Gives a method of calculating the flow distribution along a porous cylinder forming the working element of contact, filter or similar apparatus used in the chemical, metallurgical, and other industries.

We shall examine the operation of contact or filter apparatus in the form of a porous cylinder mounted coaxially inside another cylinder with solid walls (Fig. 1). The elementary flow rate of liquid or gas dQ_x through an elementary annular area df_x of the porous cylinder at a distance x from the coordinate origin, taken as the closed end of the cylinder, is obviously equal to

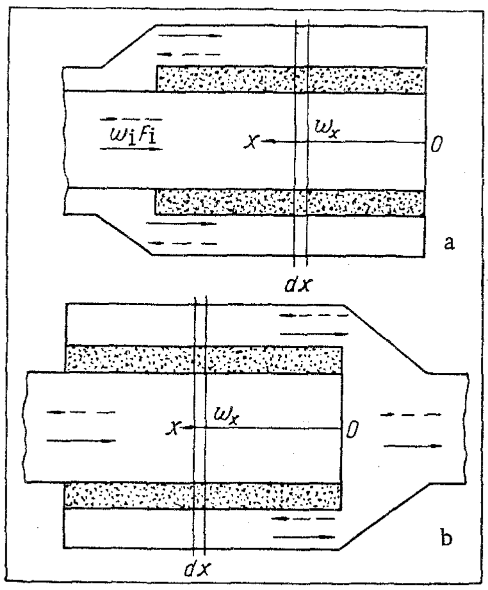

Fig. 1. Diagram of cylindrical contact apparatus: a) П-shaped; b) Z-shaped.

$$dQ_x = v_x \overline{f}\, df_x = v_x \overline{f} \pi D dx. \tag{1}$$

It is easy to show that for a coaxial arrangement of the cylinders the formula for flow through the porous surface has the form

$$v_x = \sqrt{\pm \frac{2g}{\gamma} \frac{\Delta p_x - \Delta p_x^*}{\zeta_l}} = \mu \sqrt{\pm \frac{2g}{\gamma} (\Delta p_x - \Delta p_x^*)}. \tag{2}$$

Here and henceforth the upper sign ("plus" or "minus") corresponds to the case when the direction of flow is from the inside to the outside cylinder, and the lower sign to the case when the flow is in the opposite direction.

The Bernoulli equations for the cross sections x – x and 0 – 0 of each of the cylinders (inside and outside) of a П-shaped apparatus (Fig. 1a), with allowance for the fact that at x = 0, $w_x = 0$, have the form:

$$\Delta p_x + \frac{\gamma w_x^2}{2g} = \Delta p_0 \pm \xi_C \frac{\gamma w_i^2}{2g}, \tag{3}$$

$$\Delta p_x^* + \frac{\gamma w_x^{*2}}{2g} = \Delta p_0^* \mp \xi_C^* \frac{\gamma w_x^{*2}}{2g}. \tag{4}$$

Simultaneous solution of Eqs. (1)-(4), after substitution of the dimensionless quantities $\overline{Q}_x = Q_x/Q_i$; $\overline{x} = x/L$; $\overline{S} = S_t/F_i$; $\overline{S}^* = S_t/F_i^*$, gives

$$\overline{Q}_x'^2 \pm (1 \mp \xi_C)\mu^2 \overline{f}^{-2} \overline{S}^{-2} \overline{Q}_x^2 \mp (1 \pm \xi_C^*)\mu^2 \overline{f}^{-2} \overline{S}^{*2} \overline{Q}_x^{*2} - \overline{Q}_0'^2 = 0. \tag{5}$$

It is easy to understand that the flow through the layer at any segment dx is common to both cylinders, inside and outside. Therefore in the case of a П-shaped apparatus the flow through a section x of each of the cylinders is also the same, i. e.,

$$Q_x = Q_x^*, \quad \text{or} \quad \overline{Q}_x^* = \overline{Q}_x. \tag{6}$$

Hence, after certain transformations and introduction of a correction coefficient $k_1 = 2$ (from experiment), instead of (5) we obtain

$$\bar{Q}_x'^2 \pm A_1^2 \bar{Q}_x^2 - \bar{Q}_0^2 = 0, \tag{7}$$

where

$$A_1 = 2\sqrt{(1 \mp \xi_C)\bar{S}^2 - (1 \pm \xi_C)\bar{S}^{*2}}\, \mu \bar{f}. \tag{8}$$

Integration of Eq. (7) leads to the following final design formulas:

$$\bar{w}_x = w_x/w_i = Q_x/Q_i = \mathrm{sh}(A_1 \bar{x})/\mathrm{sh}\, A_1, \tag{9}$$

$$\bar{v}_x = v_x/v_b = A_1 \,\mathrm{ch}(A_1 \bar{x})/\mathrm{sh}\, A_1, \tag{10}$$

$$\Delta \bar{p}_x = \frac{\Delta p_x - \Delta p_x^*}{\gamma w_i^2/2g} = \frac{A_1^2}{\mu^2 \bar{f}^2 \bar{S}^2} \frac{\mathrm{ch}^2(A_1 \bar{x})}{\mathrm{sh}^2 A_1}, \tag{11}$$

$$\zeta_t = \frac{\Delta p_t - \Delta p_t^*}{\gamma w_i^2/2g} = \frac{A_1}{\mu^2 \bar{f}^2 \bar{S}^2 \mathrm{th}^2 A_1} + 1 - \left(\frac{F_i}{F_i^*}\right)^2. \tag{12}$$

It is possible to consider two forms of deviation of the flow (suction) velocities along the cylindrical layer[1]:

$$\Delta \bar{v}_{max} = 1 - \frac{v_{min}}{v_b} \quad \text{and} \quad \Delta \bar{v}_{nom} = \frac{v_{max}}{v_b} - 1.$$

It is easy to show that on the basis of (10) this leads to

$$\Delta \bar{v}_{max} = A_1 \,\mathrm{th}\, A_1 - 1. \tag{13}$$

In the case of a Z-shaped apparatus the Bernoulli equations for the cylinders (Fig. 1b) are written thus

$$\Delta p_x + \frac{\gamma w_x^2}{2g} = \Delta p_0 \pm \zeta_C \frac{\gamma w^2}{2g}, \tag{14}$$

$$\Delta p_x^* + \frac{\gamma w_x^{*2}}{2g} = \Delta p_0^* + \frac{\gamma w_0^{*2}}{2g} \pm \xi_C^* \frac{\gamma w_0^{*2}}{2g} \mp \xi_C^* \frac{\gamma w_x^{*2}}{2g}. \tag{15}$$

Simultaneous solution of these equations with expressions (1) and (2) gives

$$\bar{Q}_x'^2 \pm \mu^2 \bar{f}^2 \bar{S}^2 (1 \mp \xi_C)\bar{Q}_x^2 + (1 \pm \xi_C^*)\mu^2 \bar{f}^2 \bar{S}^{*2} \bar{Q}_x^{*2} \pm (1 \pm \xi_C^*)\mu^2 \bar{f}^2 \bar{S}^{*2} - \bar{Q}_0^2 = 0. \tag{16}$$

In the case examined the flow through the outside (annular) cylinder at a distance x from the coordinate origin is equal to the difference of the flows through the initial section and the section x – x of the inside cylinder:

$$Q_x^* = Q_i - Q_x, \quad \text{or} \quad \bar{Q}_x^* = 1 - \bar{Q}_x. \tag{17}$$

Hence, after appropriate transformations,

$$\bar{Q}_x'^2 \pm A_2'^2 \bar{Q}_x^2 \pm A_2^2 \bar{Q}_x - \bar{Q}_0^2 = 0, \tag{18}$$

$$A_2' = \sqrt{(1 \mp \zeta_C)\bar{S}^2 - (1 \pm \zeta_C^*)\bar{S}^{*2}}\, \mu \bar{f}, \tag{19}$$

$$A_2 = \sqrt{2(1 \pm \zeta_C^*)}\, \mu \bar{f} \bar{S}^*. \tag{19'}$$

[1] V. N. Taliev, Aerodynamics of Ventilation [in Russian], Stroizdat, 1954.

A solution is given here only for the special case when $\xi_C = \xi_C^* = 0$ and $F_i = F_i^*$, and hence $\overline{S} = \overline{S}^*$ and $A_2^* = 0$. In this case instead of (18) we obtain

$$\overline{Q}_x'^2 \pm A_2^2 \overline{Q}_x - \overline{Q}_0'^2 = 0. \tag{20}$$

Integration of this equation leads to the following final design formulas:

$$\overline{w}_x = \overline{Q}_x = (1 \pm 0.25 A_2^2)\overline{x} \mp 0.25 A_2^2 \overline{x}^2, \tag{21}$$

$$\overline{v}_x = 1 \pm 0.25 A_2^2 \mp 0.5 A_2^2 \overline{x}, \tag{22}$$

$$\Delta \overline{v}_{max} = \Delta \overline{v}_{nom} = 0,25 A_2^2, \tag{23}$$

$$\Delta \overline{p}_x = \frac{2}{A_2^2}(1 \pm 0.25 A_2^2 \mp 0.5 A_2^2 \overline{x})^2, \tag{24}$$

$$\zeta_t = \frac{2}{A_2^2}(1 - 0.25 A_2^2)^2. \tag{25}$$

NOTATION

w_i, w_0, w_x – mean flow velocity along the inside (porous) cylinder at the initial ($x = L$) and final ($x = 0$) sections and at a section at distance x from the coordinate origin; $w_i^* = w_0^*$, w_x^* – the same for the outside (annular) cylinder; v_b, v_x – flow velocity through pores of cylinder of given diameter D, averaged with respect to the flow ($v_b = Q_i/\overline{f} S_t$) and true value at a given section x, respectively; Q_i, Q_x – flow through inside cylinder at section $x = L$ and at a distance x from coordinate origin, respectively; Q_i^*, Q_0^*, Q_x^* – same for outside cylinder; p_i, p_0, p_x – static pressures in inside cylinder at $x = L$, $x = 0$, and at a distance x from coordinate origin, respectively; p_t, p_t^* – initial total pressure in inside and outside cylinders, respectively; p_a – atmospheric pressure; $\Delta p_x = p_x - p_a$; $\Delta p_x^* = p_x^* - p_a$; $\Delta p_0 = p_0 - p_a$; $\Delta p_0^* = p_0^* - p_a$ – corresponding excess pressures; $F_i = F_x$; $F_i^* = F_x^*$; – cross-sectional areas of inside and outside cylinders, respectively; S_t – total surface of porous cylinder of given diameter; μ – coefficient of flow through layer (fabric); $\zeta l = \frac{\Delta H l}{\gamma v_b^2/2g}$ – resistance coefficient of layer (fabric) referred to mean flow rate through pores v_b; ξ_t – total resistance coefficient of entire filter (contact) element for motion from inside to outside cylinder reduced to the velocity w_i; ξ_C, ξ_C^* – resistance coefficients of cylinders proper, inside and annular respectively, reduced to velocities w_i and w_i^*; $\overline{f}$ – coefficient of clear cross section of layer (fabric); $\overline{S} = S_t/F_i$; $\overline{S}^* = S_t/F_i^*$ – relative surface of porous cylinder of given diameter D as a fraction of the cross-sectional areas of the inside and annular cylinders, respectively.

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